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Russian Translation

On the Question of the Transport of Radiant Energy in a Medium

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P. K. Konakov published a paper in this journal in 1953 in which he arrived at the following relation between the radiant energy vector and the temperature gradient in the medium [1]:

$$q_r = -\lambda_r \text{grad } \Theta \tag{1}$$

where λ_r is a proportionality coefficient, the 'radiant heat conduction coefficient'. Still earlier, S. N. Shorin used this relation for practical computations [2,3]. Keeping in mind the importance of knowing the heat transfer in a medium by means of radiation for modern engineering, let us discuss this as follows.

P. K. Konakov derived (1) on the basis of the application of the second law of thermodynamics to a non-equilibrium system. However, it is known that the use of thermodynamis to describe a non-equilibrium system is not legitimate in all cases when there is considerable deviation from equilibrium. This fact makes necessary a special investigation of the question of the applicability of (1).

In 1952, Iu. A. Surinov [4] showed in a general way that the gradient representation of the radiant energy vector is only a particular case. It appears to be correct for sufficiently small deviations from the state of thermodynamic equilibrium.

To be specific, let us analyze the simplest case - steady heat transfer through a plane layer bounded by absolutely black walls. Using the transport equation enables the establishment of the following formula for the magnitude of the directed transport of radiant energy [5]:

$$q = \iint (J^+ - J^-) \cos \Theta \, d\omega \, d\nu$$

where

$$(2)\quad J^{+}-J^{-} = \varepsilon_1 n^2 \exp\left(-\frac{1}{\cos\Theta}\int_0^x \alpha\, d\eta\right) - \varepsilon_2 n^2 \exp\left(-\frac{1}{\cos\Theta}\int_x^L \alpha\, d\eta\right)$$

$$+\frac{1}{\cos\Theta}\int_0^x \varepsilon n^2 \alpha\, \exp\left(-\frac{1}{\cos\Theta}\int_\zeta^x \alpha\, d\eta\right) d\zeta - \frac{1}{\cos\Theta}\int_x^L \varepsilon n^2 \alpha\, \exp\left(-\frac{1}{\cos\Theta}\int_x^\zeta \alpha\, d\eta\right) d\zeta$$

Here J^{+} is the absolute value of the radiation intensity in the direction from the left of the wall to the right; J^{-} correspondingly from right to left; Θ is the angle between the ray direction and the x axis (the axis is directed from the left of the wall, $x = 0$, to the right, $x = L$, perpendicularly); $d\omega$ is a solid angle element (integration is over a hemisphere); ν is the frequency; $\varepsilon_1 = \varepsilon(T_1)$ and $\varepsilon_2 = \varepsilon(T_2)$ are the radiating powers of an absolutely black body at the temperatures to the left and the right of the wall; n is the index of refraction of the medium; α is the absorption coefficient.

The first two terms in (2) correspond to the radiation energy of the wall, attenuated at the expense of its absorption in the medium; the second two describe the transport of the inherent radiation of the medium. Therefore, instead of a relation of the type of (1) we have a more complex, integral type relation. Let us show that the gradient representation (1) is obtained from it in a specific approximation. First of all, let us turn attention to the fact that the gradient representation is impossible when the first two terms in (2) exist because these terms are determined by the boundary conditions and not by the temperature distribution in the medium (if we digress from the possible dependence of α on the temperature). These terms will be sufficiently small under the conditions:

$$(3)\qquad \alpha L \gg 1$$

and that the process be analyzed sufficiently far from the wall. Here, the exponentials in the second terms will be assumed different from zero only near $\zeta = x$. This permits the use of a series expansion of εn^2 around the point x :

(4) $$\varepsilon n^2 = (\varepsilon n^2)_x + \left(\frac{\partial(\varepsilon n^2)}{\partial \zeta}\right)_x (\zeta - x) + \ldots$$

Furthermore, for simplicity, let us put $\alpha = \text{const}$. Then, neglecting the exponentials with large negative exponents, we obtain:

(5) $$J^+ - J^- = - \frac{2 \cos \theta}{\alpha} \frac{\partial(\varepsilon n^2)}{\partial x}$$

from which:

(6) $$q = - \frac{4\pi}{3\alpha} \frac{\partial}{\partial x} \int_0^\infty \varepsilon n^2 \, d\nu$$

For $n = \text{const}$, we obtain

(7) $$\lambda_r = \frac{16\pi\sigma T^3 n^2}{3\alpha}$$

where σ is the Stefan-Boltzmann constant. Therefore, in this case, we really obtain the gradient relation between the flow of energy and the temperature. We found (6) before by another method [5] and it has been found to correspond with the results of investigations by other authors [6,7]. S. N. Shorin arrived at the same type of relation by starting from the representation of the radiant energy as a photon gas and using the gradient relation (1).

Formula (6), derived here as a particular case, is correct for strongly-absorbing media but ceases to be true for weakly-absorbing media. Actually, it is sufficient to assume that temperature gradients exist in transparent media at the expense of the process of molecular heat conduction since $q \rightarrow \infty$ is obtained from (6). An investigation of a relation of type (2) would permit us to establish that the thermal flow in weakly-absorbing media appears to be linearly dependent on the magnitude of the absorption coefficient [5]. The results of [6,7] are found to agree with this. The question of the relation of the thermal radiation flow to the temperature distribution in the medium is similar, to a great extent, to the same question for molecular heat conduction. The gradient relation of the flow to the temperature distribution (Fourier law) appears to be correct only for small deviations from system equilibrium, when the free path length is very small compared with the system dimensions. In the case when they are comparable, the heat flow is essentially dependent on the character of the process at the walls and its relation to the temperature distribution is given by an integral relation.

The H - theorem, used to find the shape of the molecule distribution function, appears to be correct for the case of small deviations from equilibrium, when the Fourier law is correct. In the general case, this is not so. The result of all the above is the statement that the gradient representation of the radiant energy vector is competent in the particular case of a strongly-absorbing medium.

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